Scientific Review – Engineering and Environmental Sciences (2019), 28 (4), 569–583
Sci. Rev. Eng. Env. Sci. (2019), 28 (4)
Przegląd Naukowy – Inżynieria i Kształtowanie Środowiska (2019), 28 (4), 569–583
Prz. Nauk. Inż. Kszt. Środ. (2019), 28 (4)
http://iks.pn.sggw.pl
DOI 10.22630/PNIKS.2019.28.4.52

Volodymyr HUD[1], Oleg PINCHUK[2], Petro MARTYNIUK[1], Ievgenii GERASIMOV[3], Pavlo VOLK[2]

[1]Institute of Automatics, Cybernetics and Computer Engineering
[2]Institute of Water Management and Environmental Engineering
[3]Research Department
National University of Water and Environmental Engineering – NUWEE

Mathematical modelling of heat transfer in a greenhouse with surface soil heating system*

Key words: modelling, heat transfer, greenhouse, soil heating, sleeve cover

Introduction

The problem of getting early and sustainable yields, particularly of vegetables and berries, is one of the major social and economic issues in agriculture of many countries. Climate change and weather conditions that become unpredictable further intensify the urgency of this issue. In the generally accepted climate change trend, experts pay regard even to the possibility of local cold snaps' occurrence in certain areas, prolonged cold periods in spring, significant fluctuations of weather conditions (Boichenko, 2005; Iglesias & Garrote, 2015; Rokochynskiy, Volk, Frolenkova, Prykhodko, Gerasimov & Pinchuk, 2019; Rokochynskiy, Volk, Pinchuk, Turcheniuk, Frolenkova & Gerasimov, 2019).

The study of the impact of possible climate and weather changes on the development and productivity of crops and the development of measures to prevent negative climatic and weather phenomena become relevant not only for agricultural scientists but also for practitioners.

Heat resources play a significant role in yield formation. Each representative of the plant kingdom has its own, strictly individual thermal and temporal structure of development cycle from sowing to harvesting. The cycle parameters are strictly related to the corresponding level of temperature in soil and air. Therefore, the purposeful control

*Due to complexity of the article text was formatted in one-colunm page style.

of the plant ontogenesis is possible by managing the thermal regime of the microenvironment of its habitat. It is obvious that the thermal regime of soil and ground air layer will also be determinative for the intensification of growth and early growing of crops (Bita & Gerats, 2013; Pinchuk, 2015; Rotjanakusol & Laosuwan, 2018).

Warm wastewater (heat exchange water) from industrial and power facilities with a temperature of 20–40°C, as well as geothermal water can be attractive sources of free heat energy. Their attractiveness lies in the fact that such temperatures are optimal for the development of most representatives of plant kingdom.

One of the most promising and effective ways of heating the soil may be its surface heating by warm water. Technologically, this technique can be implemented by directing the flow of water along the surface of the soil, but technically by using special cover sleeves (Fig. 1) in greenhouses (Pinchuk, 2012; 2015).

FIGURE 1. General view of greenhouse with cover sleeves

In this case, the use of surface soil heating with water-filled cover sleeves requires solving a number of scientific and applied problems. In particular, there is a need for the rationale and development of rational forms and structures of cover sleeves, system structures on their basis, structures of temporary portable transparent shelters, means of automation and regulation of water regime of soil and ground air layer, the definition of rational hydraulic and heat engineering operating modes, the justification of design techniques, installation, operation and maintenance of surface soil heating systems, and methods of growing various crops in soil ameliorated by heat.

Accordingly, the overall aim of the paper is to develop a mathematical model of heat exchange of a greenhouse with a system of surface soil heating.

Material and methods

Mathematical model

The design peculiarity of such a greenhouse is the availability of different methods of heat transfer between the system's environments – thermal conductivity, convection, to some extent – radiation. Moreover, there are continuous connections between them and they are determined by the structural system peculiarities (geometric dimensions of the elements, thermal and physical characteristics of the materials used, etc.).

The authors described heat transfer processes in the greenhouse on the basis of the convective heat transfer equation (Vostrikov, Romaniuk, Pinchuk & Vostrikova, 2008; Pryor, 2011; Mayboudi, 2018):

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) - \rho C_p (\boldsymbol{u} \cdot \nabla T) + Q \tag{1}$$

where:
ρ – air density;
C_p – specific heat capacity of air;
T – temperature;
t – time;
λ – thermal conductivity coefficient;
$\boldsymbol{u}$ – velocity vector;
Q – energy obtained by insolation.

The determination of the distribution of air velocities in the greenhouse was carried out using a solution of the Navier–Stokes equation (Pryor, 2011):

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{F} - grad\ \ p + (\mu_V + \frac{\mu}{3}) grad\ \ div\ \boldsymbol{u} + \mu \nabla^2 \boldsymbol{u} \tag{2}$$

where:
$\boldsymbol{F}$ – force vector determined by dependence: $\boldsymbol{F} = \rho \boldsymbol{a}$ (3), where $\boldsymbol{a}(x, y, z) = (0; 0; -g)$ is an acceleration vector;
p – pressure;
μ – dynamic viscosity;
μ_V – bulk viscosity;

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ – the Laplace operator.

The dependence of the air density on pressure and temperature was represented by the equation of state for ideal gases:

$$\rho = \frac{pM}{RT} \tag{4}$$

where:
M – molecular weight;
R – gas constant.

The integrity of the air flow is described in accordance with the following:

$$\frac{\partial \rho}{\partial t} + \rho \cdot div\ \boldsymbol{u} = 0 \qquad (5)$$

In the Cartesian coordinate plane projections, the relations (1)–(5) will look like this:

$$\begin{cases} \rho C_p \frac{\partial T}{\partial t} = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) - \rho C_p (u_x \frac{\partial T}{\partial x} - u_y \frac{\partial T}{\partial y} - u_z \frac{\partial T}{\partial z}) + Q \\ \rho \frac{\partial u_x}{\partial t} = \frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left(\mu \frac{\partial u_x}{\partial x}\right) + \frac{\partial}{\partial x}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right] \\ \rho \frac{\partial u_y}{\partial t} = -\rho g - \frac{\partial p}{\partial y} + 2\frac{\partial}{\partial y}\left(\mu \frac{\partial u_y}{\partial y}\right) + \frac{\partial}{\partial y}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)\right] \\ \rho \frac{\partial u_z}{\partial t} = \frac{\partial p}{\partial z} + 2\frac{\partial}{\partial z}\left(\mu \frac{\partial u_z}{\partial z}\right) + \frac{\partial}{\partial z}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)\right] \\ \frac{\partial \rho}{\partial t} = -\left(\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z}\right) \\ \rho = \frac{pM}{RT} \end{cases} \qquad (6)$$

where $\boldsymbol{u}$ is velocity vector with projections u_x, u_y, u_z on the corresponding coordinate axes x, y, z.

In the general case, the system of equations solution (6) is a rather complex and time-consuming task (Marsden & Chorin, 2004; Samarskii & Vabishchevich, 2009). Therefore, as a rule, for a problem of this kind, there is a need to take some simplifications and assumptions that allow solving it in an approximate manner.

Taking into account the factors listed above, the solution to the system of equations (6) was obtained by the finite element method (Pryor, 2011; Pepper & Heinrich, 2017).

The authors analysed heat transfer processes that occur in the greenhouse with radius (R), where there are flexible cover sleeves with elliptical-shaped cross-section, as well as soil sections of depth (H) and width (L), by computer simulation of a two-dimensional problem (Fig. 2).

We assumed that the projections of the vector values on z-axis and the corresponding first-order and second-order derivatives along z are equal to zero, therefore:

$$u_z = 0 \qquad (7)$$

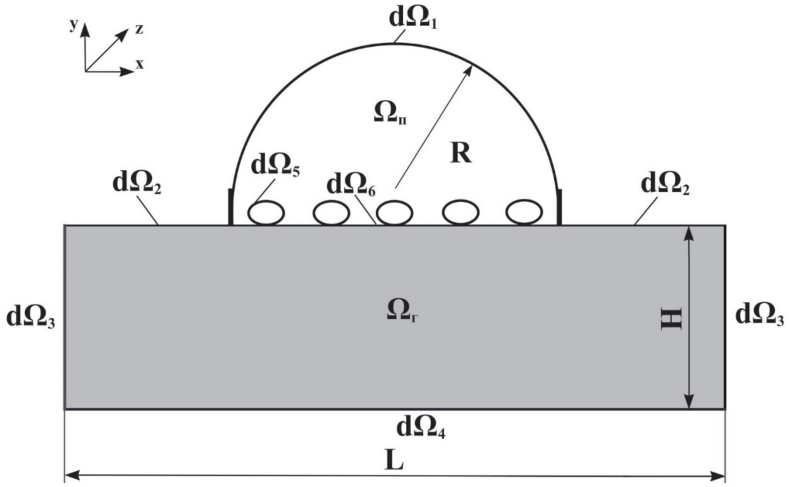

FIGURE 2. Model of greenhouse with five cover sleeves

$$\frac{\partial(\rho u_z)}{\partial z}=0;\quad \frac{\partial u_z}{\partial x}=0;\quad \frac{\partial u_z}{\partial y}=0;\quad \frac{\partial u_z}{\partial t}=0 \tag{8}$$

$$\frac{\partial p}{\partial z}=0;\quad \frac{\partial u_z}{\partial z}=0;\quad \frac{\partial u_x}{\partial z}=0;\quad \frac{\partial u_y}{\partial z}=0 \tag{9}$$

$$\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)\right]=0;\quad \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right)\right]=0 \tag{10}$$

$$\frac{\partial}{\partial z}\left(\mu\frac{\partial u_z}{\partial z}\right)=0;\quad \frac{\partial}{\partial z}\left[\left(\mu_V-\frac{2}{3}\mu\right)div\ u\right]=0 \tag{11}$$

$$\frac{\partial^2 T}{\partial z^2}=0 \tag{12}$$

Taking into account the above-mentioned data, the system of equations (6) becomes:

$$\begin{cases} \rho C_p \frac{\partial T}{\partial t} = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - \rho C_p (u_x \frac{\partial T}{\partial x} - u_y \frac{\partial T}{\partial y}) + Q \\ \rho \frac{\partial u_x}{\partial t} = \frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left(\mu \frac{\partial u_x}{\partial x}\right) + \frac{\partial}{\partial x}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] \\ \rho \frac{\partial u_y}{\partial t} = -\rho g - \frac{\partial p}{\partial y} + 2\frac{\partial}{\partial y}\left(\mu \frac{\partial u_y}{\partial y}\right) + \frac{\partial}{\partial y}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] \\ \frac{\partial \rho}{\partial t} = -\left(\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y}\right) \\ \rho = \frac{pM}{RT} \end{cases} \tag{13}$$

Given the slow change of the system parameters in time, we solve it for stationary mode, in which:

$$\frac{\partial T}{\partial t} = 0;\ \frac{\partial u_x}{\partial t} = 0;\ \frac{\partial u_y}{\partial t} = 0;\ \frac{\partial \rho}{\partial t} = 0 \tag{14}$$

Taking into account (14) the system of equations (13) changes and becomes:

$$\begin{cases} \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - \rho C_p (u_x \frac{\partial T}{\partial x} - u_y \frac{\partial T}{\partial y}) + Q = 0 \\ \frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left(\mu \frac{\partial u_x}{\partial x}\right) + \frac{\partial}{\partial x}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] = 0 \\ -\rho g - \frac{\partial p}{\partial y} + 2\frac{\partial}{\partial y}\left(\mu \frac{\partial u_y}{\partial y}\right) + \frac{\partial}{\partial y}\left[\left(\mu_V - \frac{2}{3}\mu\right) div\ u\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)\right] = 0 \\ \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} = 0 \\ \rho = \frac{pM}{RT} \end{cases} \tag{15}$$

We set out boundary conditions on the border of the calculated model area (Fig. 2) on the basis of the following assumptions:

1. The temperature on the surface of the soil outside the greenhouse and on the surface of the film is equal to ambient temperature:

$$T\big|_{d\Omega_1} = T_e\,;\; T\big|_{d\Omega_2} = T_e \quad (16)$$

2. At a distance of $\frac{L}{2} = 10 \cdot R$ from the greenhouse centre, the heat flow through the lateral surface of the soil is considered to be equal to zero:

$$\lambda_{s.} \frac{\partial T}{\partial x}\bigg|_{d\Omega_3} = 0 \quad (17)$$

3. In accordance with (Pinchuk, 2012), at a depth $H = H_0$, the constant soil temperature is maintained:

$$T\big|_{d\Omega_4} = T_0 \quad (18)$$

4. On the cover sleeve surface the temperature is equal to temperature of warm water:

$$T\big|_{d\Omega_5} = T_{ww} \quad (19)$$

5. In the greenhouse, on the boundary layer of air next to soil, the condition of equality of heat flows is fulfilled:

$$\lambda_{s.shel.} \frac{\partial T}{\partial y}\bigg|_{d\Omega_6} = -\lambda_{a.shel.} \frac{\partial T}{\partial y} \quad (20)$$

6. The air velocity on the inner surface of the film, on the surface of the cover sleeves and on the surface of the soil in the greenhouse is zero:

$$u\big|_{d\Omega_1} = 0;\; u\big|_{d\Omega_5} = 0;\; u\big|_{d\Omega_6} = 0 \quad (21)$$

Experimental research

The research covered the model of the greenhouse with a radius R = 0.5 m and the shelter framework height h = 0.2 m. The scholars also assumed that H_0 = 8.6 m and $L = 20R$ = 10.0 m.

The dependence of specific heat capacity and thermal conductivity on soil moisture was approximated by the following linear equations (Kurtener & Chudnovsky, 1969; 1979):

$$\lambda = \lambda_0 + \alpha\omega \quad (22)$$

$$c = c_0 + \beta\omega \quad (23)$$

where:
$\lambda_0 = 0.420$ W·m^{-1}·K^{-1}, $c_0 = 780.0$ J·kg^{-1}·K^{-1} – thermal conductivity and specific heat capacity of dry soil;
$\alpha = 0.022$ W·m^{-1}·K^{-1}·%$^{-1}$, $\beta = 23.33$ J·kg^{-1}·K^{-1}·%$^{-1}$ – coefficients taking into account thermal conductivity and specific heat capacity change when the soil moisture is changed by 1%.

Given that the soil moisture can vary from 0 to 40%, the authors assumed the values of thermal conductivity (λ) and heat capacity (c) for a moisture content of 20%, i.e. $c = 1{,}246.6$ J·kg^{-1}·K^{-1} and $\lambda = 0.860$ W·m^{-1}·K^{-1} respectively.

The researchers carried out the simulation for ambient temperatures (from –20 to +20°C), and for the temperature of warm water in the sleeves between +15 and +35°C. Due to the variability of the insolation level of solar radiation, the simulation was realized at $Q = 0$.

In order to verify the correctness of the model, the scientists conducted experimental studies for a greenhouse with a size of 1 × 6 m. In a field experiment, the authors used a system of five cover sleeves (6.5-meter long), combined in a section using distributing and accumulating collectors (Fig. 3), for heating the soil in a greenhouse (one-meter wide). They used a 100-micron polyethylene film as a cover material. The temperature of the heat-carrying medium varied from 17 to 27°C.

The experimental system worked as follows: under pressure of 0.1 m, through pipeline (2), the heat-carrying medium from the warm water reservoir 1 enters the distributing collector, which distributes water between the sleeves (4). Warm wa-

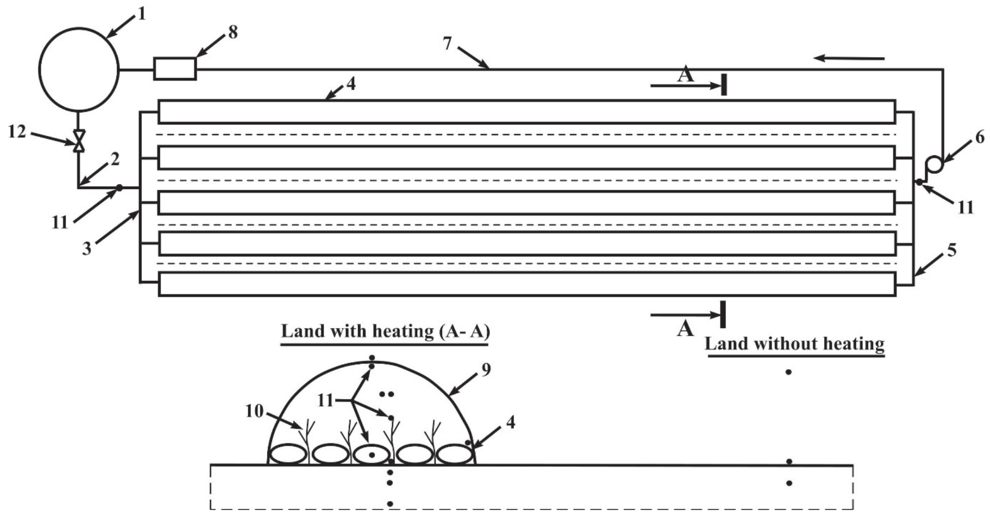

FIGURE 3. Field experiment design: 1 – warm water reservoir; 2 – heat supply pipeline; 3 – distributing collector; 4 – cover sleeve; 5 – accumulating collector; 6 – pump unit; 7 – conveyor tube; 8 – electric heater; 9 – tunnel cover made of polyethylene film (greenhouse); 10 – plants; 11 – temperature sensors; 12 – slide valve

ter passing through cover sleeves heats the soil and surface layer of air around the plants, intensifying the processes of their growth and development. The cooled water enters the suction line of the pump unit (6) through the collector (5) and then flows through the conveyor tube (7) to the electric heater (8). In the electric heater (8), the coolant is heated to the specified temperature and enters the warm water reservoir (1). The flow rate of the heat-carrying medium from the reservoir 1 is controlled by the valve (12), afterwards the cycle is repeated. In the heating system, the water temperature was kept within the range of 20–30°C.

In order to ensure the continuity of measuring temperature parameters in the environment "soil – plant – air-ground interface", an increase in the number of measurements carried out with constant high frequency the researchers applied specially designed automated data collection system.

The authors measured the soil temperature in an automated mode at the air-ground heating area in the centre of the greenhouse at depths of 0, 10, 20, 40 cm, as well as the air temperature in the shelter at heights of 20, 30, 70 cm. They also measured the air temperature in the shelter at the side of the film at a height of 20 cm from the soil surface and outside of the shelter above the film. The temperature of the soil outside the shelter was measured at a depth of 0 and 20 cm, and the air – at a height of 200 cm. Measurement and logging of temperature parameters was made once a minute, and data commit process took place on a personal computer at five-minute intervals.

The research team used strawberries of early-bearing Italian cultivar 'Clyro' as the main agricultural crop, the indicator, which planting is done at a row distance of 20 cm and plant distance of 15 cm.

Results and discussion

As a result of the simulation for the warm water temperature T_{ww} = 20°C and the air temperature T_a = 0°C, it turned out that the highest temperature on the soil surface (Fig. 4) is at the points of contact between the water sleeves and the ground. At the same time, there are local temperature minimal on the soil surface between the sleeves in which the temperature is 4–6°C lower than the maximum value. The soil temperature decrease from the centre to the edge of the sleeve indicates significant losses of thermal energy through the soil on the greenhouse boundary. With increasing depth, the temperature distribution has a monotonic character with a clear single maximum in the middle of the greenhouse.

As a result of the analysis of the air temperature distribution in the greenhouse (Fig. 5), it turned out that at the sleeves' surface level, it periodically changes with a maximum precisely on the sleeves' surface. At the same time, between the sleeves, the air temperature is 2–6°C lower than the sleeve surface with a rapid decrease near the surface of the shelter film. As the height from the soil surface increases, the air temperature in the shelter decreases, and its distribution, unlike the soil, contains the expressed local extrema in the shelter centre.

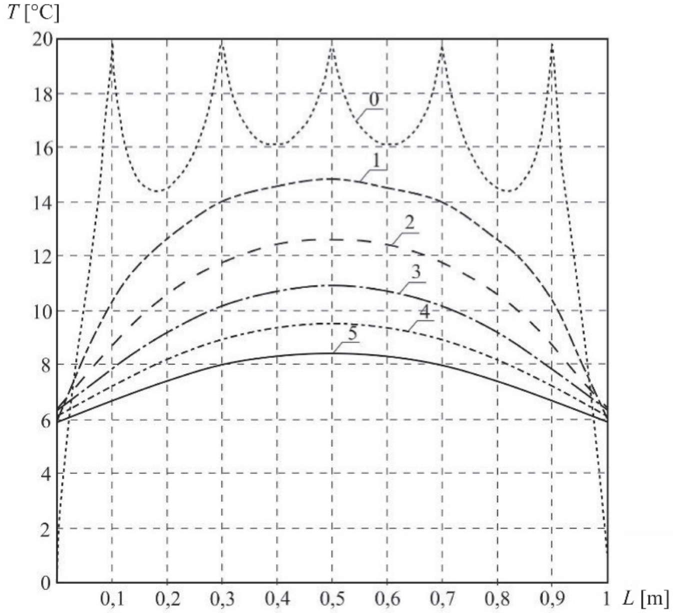

FIGURE 4. Soil heating pattern in a greenhouse with soil heating system at $T_e = 0°C$, $T_w = 20°C$: 0 – on the soil surface; 1 – at a depth of 0.1 m; 2 – at depth 0.2 m; 3 – at a depth of 0.3 m; 4 – at a depth of 0.4 m; 5 – at a depth of 0.5 m

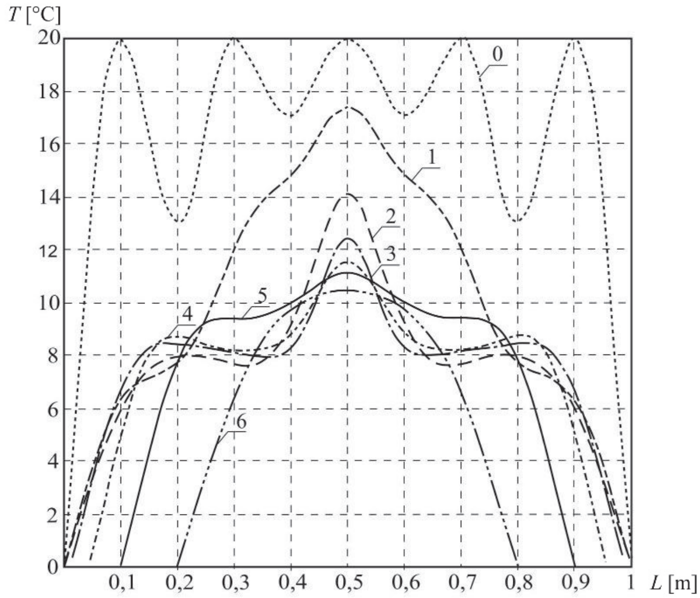

FIGURE 5. Air heating pattern in a greenhouse with soil heating system at $T_e = 0°C$, $T_w = 20°C$: 0 – at a height of sleeves' surface; 1 – at a height of 0.1 m; 2 – at a height of 0.2 m; 3 – at a height of 0.3 m; 4 – at a height of 0.4 m; 5 – at a height of 0.5 m; 6 – at a height of 0.6 m

As a result of the analysis of air circulation in the greenhouse (Fig. 6), it turned out that the reason for the expressed air temperature maximum in the centre of the greenhouse is the presence of vertical convection due to the high temperature gradient between the surface of the sleeve and the surface of the shelter and the gradual cooling of the air while lowering near the surface of the shelter.

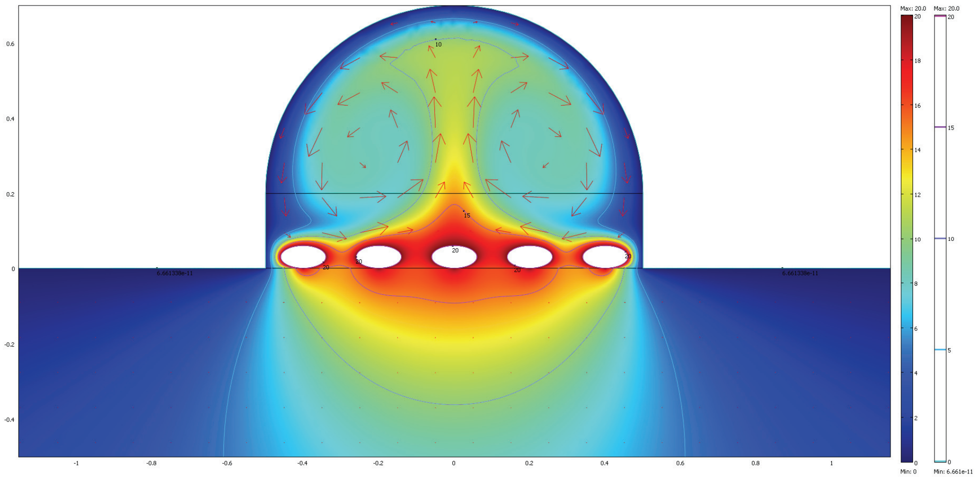

FIGURE 6. Results of computer simulation of temperature distribution in a greenhouse with soil heating system at warm water temperature $T_{ww} = 20°C$ and air temperature $T_a = 0°C$

In order to check the mathematical model adequacy and its possibilities for predicting the temperature regime of the ground air and soil layer in the greenhouse the team compared the obtained distribution with the results of experimental studies. Aiming to reduce the influence of solar insolation, measurements were made at night. As a result of temperature distribution comparison (Fig. 7), it was found that the deviation between the experimental research results and the calculated values was not more than 8%.

Conclusions

Mathematical models of heat transfer and heat exchange form the theoretical basis for substantiating, designing engineering processes and technical means of heating the soil in a greenhouse. When designing mathematical models of heat transfer and heat exchange in the system "soil-plant-surface air layer" it is necessary to take into account the main factors that determine heat exchange and heat transfer processes.

As a result of the extensive theoretical studies, the authors developed a mathematical model of the thermal regime of the greenhouse, taking into account the air convection in its volume. This mathematical model is a system of equations,

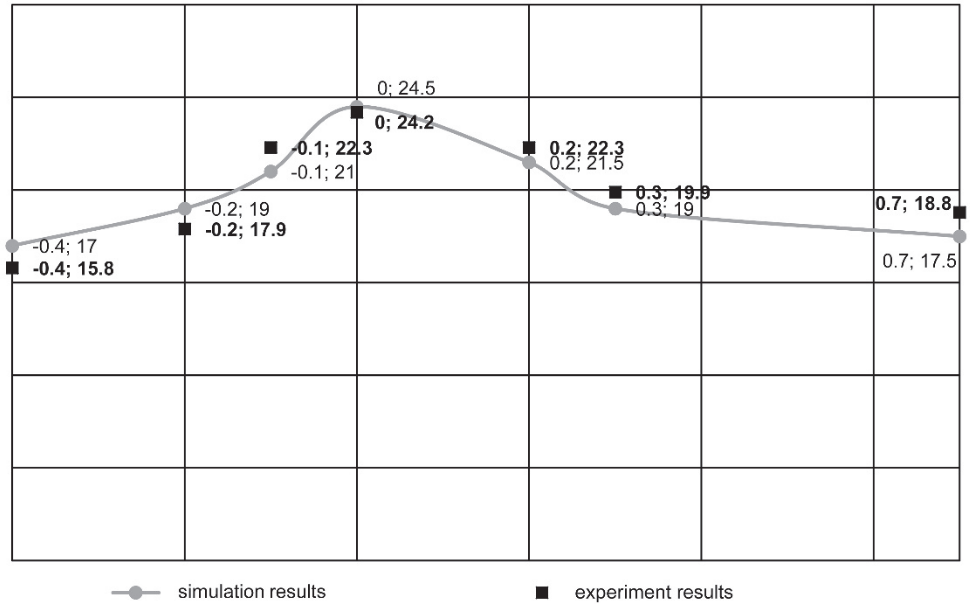

FIGURE 7. Comparison of the numerical analysis results and field experiments in terms of soil and air temperature values in a greenhouse with soil heating system at a temperature of 25°C

which consists of the thermal balance equation with regards to thermal conductivity and convection component, the Navier–Stokes equation and the flow continuity equation.

The theoretical calculations established sufficiently high heat amelioration effects in the greenhouse soil and air under the conditions of surface heating by cover sleeves. A case study simulating the temperature of 0°C and the warm water temperature of 20°C confirmed the creation of the area of guaranteed heating in the air under the cover, and heat penetration of the soil up to a depth of 40 cm.

The research proved that air circulation has a significant influence on the temperature distribution, and this impact results in maximum temperature in the centre of the greenhouse. The analysis established that the temperature of the soil under the shelter decreases from the middle to its edge by 2–6°C, depending on the distance to its surface.

Comparison of the results of experimental studies with the results of mathematical modelling showed that the suggested mathematical model allows predicting the thermal regime in greenhouses with surface soil heating by cover sleeves with a high degree of reliability.

The proposed methodology for modelling heat transfer and heat exchange processes can be applied to different designs of greenhouses and allows predicting their efficiency under anticipated soil and climatic conditions.

References

Bita, C. & Gerats, T. (2013). Plant tolerance to high temperature in a changing environment: scientific fundamentals and production of heat stress-tolerant crops. *Frontiers in Plant Science*, *4*, 273. https://doi.org/10.3389/fpls.2013.00273

Boichenko, S. (2005). *Hlobalni i rehionalni kolyvannia (zminy) klimatu ta mozhlyvi ekolohichni naslidky vid nykh na terytorii Ukrainy: napivempirychni modeli, stsenarii* [*Global and regional climate fluctuations (changes) and their possible ecological consequences on the territory of Ukraine: semiempirical models, scenarios*] (doctoral dissertation). Odessa.

Iglesias, A. & Garrote, L. (2015). Adaptation strategies for agricultural water management under climate change in Europe. *Agricultural Water Management*, *155*, 113-124. https://doi.org/10.1016/j.agwat.2015.03.014

Kurtener, D. & Chudnovsky, A. (1969). *Raschet i regulirovanie teplovogo rezhima v otkrytom i zashchishchennom grunte* [*Calculation and control of the thermal regime in open and covered soil*]. Leningrad: Hydrometeoizdat.

Kurtener, D. & Chudnovsky, A. (1979). *Agrometeorologicheskie osnovy teplovoielioratsii* [*The agrometeorological foundation of thermal melioration of soil*]. Leningrad: Hydrometeoizdat.

Marsden, J.E. & Chorin, A. (2004). *Matematicheskie osnovy mekhaniki zhidkosti* [*A mathematical introduction to fluid mechanics*]. Moskva: Librokom.

Mayboudi, L. (2018). *Heat transfer modelling using COMSOL: Slab to radial fin*. Dulles, USA: Mercury Learning.

Pepper, D.W. & Heinrich, J.C. (2017). *The Finite Element Method: Basic Concepts and Applications with MATLAB, MAPLE, and COMSOL, Third Edition*. Boca Raton, FL: CRC Press. https://doi.org/10.1201/9781315395104

Pinchuk, O. (2012). *Obhruntuvannia konstruktsii ta parametriv hidrotekhnichnoi systemy poverkhnevoho obihrivu hruntu obolonkamy-rukavamy pry vykorystanni skydnykh teplykh vod* [*Justification of design and parameters of hydrotechnical surface soil heating system using cover sleeves by means of warm waste water*] (candidate dissertation). Rivne.

Pinchuk, O. (2015). Vozmozhnosti teplovoi melioratsii dlia obespecheniia prodovolstvennoi bezopasnosti strany [Possibilities of heat amelioration to ensure the country's food safety]. *Prirodoobustroistvo: nauchno-prakticheskii zhurnal* [*Natural Habitat: Scientific and Practical Journal*], *1*, 6-11.

Pryor, R.W. (2011). *Multiphysics modeling using COMSOL?: A first principles approach*. Burlington, MA: Jones & Bartlett Learning.

Rokochynskiy, A., Volk, P., Frolenkova, N., Prykhodko, N., Gerasimov, I. & Pinchuk, O. (2019). Evaluation of climate changes and their accounting for developing the reclamation measures in western Ukraine. *Scientific Review Engineering and Environmental Sciences*, *28*(1), 3-13. https://doi.org/10.22630/PNIKS.2019.28.1.1

Rokochynskiy, A., Volk, P., Pinchuk, O., Turcheniuk, V., Frolenkova, N. & Gerasimov, I. (2019). Forecasted estimation of the efficiency of agricultural drainage on drained lands. *Journal of Water and Land Development*, *40*(1-3), 149-153. https://doi.org/10.2478/jwld-2019-0016

Rotjanakusol, T. & Laosuwan, T. (2018). Estimation of land surface temperature using Landsat satellite data: A case study of Mueang Maha Sarakham District, Maha Sarakham Province, Thailand for the years 2006 and 2015. *Scientific Review – Engineering and Environmental Sciences*, *27*(4), 401-409. https://doi.org/10.22630/PNIKS.2018.27.4.39

Samarskii, A. & Vabishchevich, P. (2009). *Chislennye metody resheniia zadach konvektsii-diffuzii* [*Numerical methods for solving convection-diffusion problems*]. Moskva: Librokom.

Vostrikov, V., Romaniuk, I., Pinchuk, O. & Vostrikova, N. (2008). Modeliuvannia teplovykh protsesiv v hrunti pry obihrivi vodonapovnenymy rukavamy [Modelling thermal processes in soil while heating with water-filled sleeves]. *Melioratsiia i vodne hospodarstvo: mizhvidomchyi tematychnyi naukovyi zbirnyk* [*Reclamation and Water Engineering: Interagency Thematic Collection of Research Papers*], *96*, 244-253.

Summary

Mathematical modelling of heat transfer in a greenhouse with surface soil heating system. The use of low-temperature heat of industrial and natural origin for heating the soil in greenhouses allows practitioners to get very early vegetable and berry crops. The paper suggests a mathematical model of greenhouse heat exchange with a system of soil surface heating for substantiating the system structure and its efficiency in different conditions. The solution of the mathematical model was performed using the method of least squares in COMSOL Multiphysics software. The comparison of the results of experimental studies with the results of mathematical modelling revealed that the proposed mathematical model with a high degree of reliability allows predicting the thermal regime in greenhouses with surface soil heating using cover sleeves.

Authors' address:
Volodymyr Hud
(https://orcid.org/0000-0002-0355-3984)
National University of Water and Environmental Engineering
Institute of Automatics, Cybernetics and Computer Engineering
Department of Automation, Electrical and Computer-Integrated Technologies
11 Soborna, 33028 Rivne
Ukraine
e-mail: v.m.hud@nuwm.edu.ua

Oleg Pinchuk
(https://orcid.org/0000-0001-6566-0008)
National University of Water and Environmental Engineering
Institute of Water Management and Environmental Engineering
Department of Hydroinformatics
11 Soborna, 33028 Rivne
Ukraine
e-mail: o.l.pinchuk@nuwm.edu.ua

Petro Martyniuk
National University of Water and Environmental Engineering
Institute of Automatics, Cybernetics and Computer Engineering
Department of Applied Mathematics
11 Soborna, 33028 Rivne
Ukraine
e-mail: p.m.martyniuk@nuwm.edu.ua

Ievgenii Gerasimov
(https://orcid.org/0000-0001-7194-8251)
National University of Water and Environmental Engineering
Research Department
11 Soborna, 33028 Rivne
Ukraine
e-mail: e.g.gerasimov@nuwm.edu.ua

Pavlo Volk
(https://orcid.org/0000-0001-5736-8314)
National University of Water and Environmental Engineering
Institute of Water Management and Environmental Engineering
Department of Water Engineering and Water Technology
11 Soborna, 33028 Rivne
Ukraine
e-mail: p.p.volk@nuwm.edu.ua