

Scientific Review – Engineering and Environmental Sciences (2020), 29 (4), 517–531
Sci. Rev. Eng. Env. Sci. (2020), 29 (4)
Przegląd Naukowy – Inżynieria i Kształtowanie Środowiska (2020), 29 (4), 517–531
Prz. Nauk. Inż. Kszt. Środ. (2020), 29 (4)
<http://iks.pn.sggw.pl>
DOI 10.22630/PNIKS.2020.29.4.45

Iqbal Khalaf Al-ATABY¹, Amani I. Al-TMIMI²

¹Mustansriyah University, Collage of Science

²Alkarkh University of Science, Energy and Environment Sciences Collage

Estimate the probability density function of maximum temperature for the Middle East

Key words: maximum temperature, probability distribution, Middle East

Introduction

Since 1950 the number of heat waves worldwide has increased, and heat waves have become longer. The hotter days and nights have become hotter more frequent. In the past several years the global area hit by extremely unusual hot summer-time temperatures has increased (Steffen, Hughes & Perkins, 2014). A heat wave is generally defined as a period of several days to weeks of abnormally hot weather. The National Weather Service (NWS) defines a heat wave as “a period of abnormally hot weather, uncomfortable and unusually humid”, and usually takes two or more days (Karl, Melillo & Peterson, 2009). In recent years, the Middle East region has witnessed a noticeable rise in temperatures (Brown &

Crawford, 2009), which clearly affects the daily behavior of the human being and his health in addition to its effects on plants, which led to the expansion of desertification and the deterioration of the vegetation cover, as well as affecting the food supply, water resources, and ecosystems. Hence, the increase of temperature average is considered one of the signs of climate change, which is one of the causes of environmental pollution so it is expected that heat wave incidence and intensity continue.

The Middle East spans several climates zones, where the eastern part is Mediterranean, the northern part is considered to be subtropical while the Arabian Peninsula has a hyper-arid desert climate. Different climate zones are found in the various mountain areas. The region has among the hottest and driest conditions found in the world. Numerous heat records were in the Middle East in the summer of 2010 with temperatures

reaching 52.0°C in Jeddah (WMO2011) (Brown & Crawford, 2009).

The first region-wide trend analysis of the Middle East extreme indices observed in the period 1950–2003 at 52 stations covering almost 15 countries was reported by Zhang et al. (2005). The results observed from this study shows statistically significant and spatially reasonable trends in temperature indices, which clearly indicate temperature increase in the region. Analysis of long-term temperature data suggests that since the 1970s the frequency of heat extremes has increased in the MENA (Tanarhte, Hadjinicolaou & Lelieveld, 2015). Simulating the climate of the region is a challenge for climate models (Evans, Smith & Oglesby, 2004), due in part to the high natural inter-annual variability, the topography of the region which includes multiple mountain ranges and inland seas, and the presence of a slight cooling trend in recent decades despite the global trend being a warming. This paper attempts to determine the most suitable probability distribution of daily maximum temperature and check the accuracy of the fitted probability distributions using the goodness of fit criteria.

Middle east climate

Middle Eastern climatic conditions vary greatly, depending on the season and the geography, but simply can be characterized in two words: hot and dry, although winters are mild with some rain. Is a region that spans southwestern Asia, western Asia, and northeastern Af-

rica. Although much of the Middle East region has a Mediterranean climate type, with wet winters and dry summers. Although the hot arid, or desert, climate predominates in the region, the well-watered highlands of Turkey and the mountains of Iran and Ethiopia are important as sources of the region's major rivers. The Middle East is as one of the regions most affected by dust, in the world, next to Africa. Dust or sand storms are caused by the outflow from low-pressure cells passing through a desert area from West to East. Sand storms can occur throughout the year in the Middle East, but the prime months are May–September.

The climate of the Middle East ranges from the warm summers and cold winters of highland Turkey and Iran, through hotter summers and cool winters of northern Mesopotamia and the Mediterranean coast, to the extreme temperatures of the Arabian Desert. Most, but not all, of the region is arid (Hasanean, 2015).

Study area

Middle East region located approximately between 10° and 40° latitudes and with 20° and 65° of longitude line approximately, and this region located basically southwestern Asia and northeastern Africa. It extends over 2,000 miles from the Black Sea in the northern region of the Arabian Sea in the South, and about 1,000 miles from the Mediterranean Sea in the West to the mountains of Iran. The Middle East domain used in this study is shown in Figure 1.



FIGURE 1. The Middle East

Material and methods

Data

The data used in this study are monthly daily mean of maximum temperature at height of 2 m covered Middle East as a grid of 1,581 points extends from 15–65° N latitudes and 15–45° E longitudes with a uniform grid interval of 1.5° longitude and 1.5° latitude for selected months (March, April, May) represent spring and selected months (June, July, August) represent summer for the period 1979–2018, at the time 00.00 UTC, from the European Centre for Medium-Range Weather Forecasts (CMWF), model ERA-interim. This model is characterized by providing data with clarity degree (0.75° × 0.75°) by having two forms of data (full resolution and low resolution) in the first type, the amount of spatial accuracy of the data

can be controlled, while the second type is the degree of clarity as shown in Figure 2 (Paul, 2013).

Probability distributions

Many probability distribution functions (PDFs) have been proposed in recent past years, but in this paper logistic, Rayleigh and gamma distribution are used to describe the characteristics of maximum temperature.

Logistic distribution

The logistic distribution is a continuous probability distribution. Its distribution function is the logistic function, which appears in logistic regression and feed forward neural networks. It resembles the normal distribution in shape but has heavier tails. Its probability density function is given as (Al-Kadim, 2011):

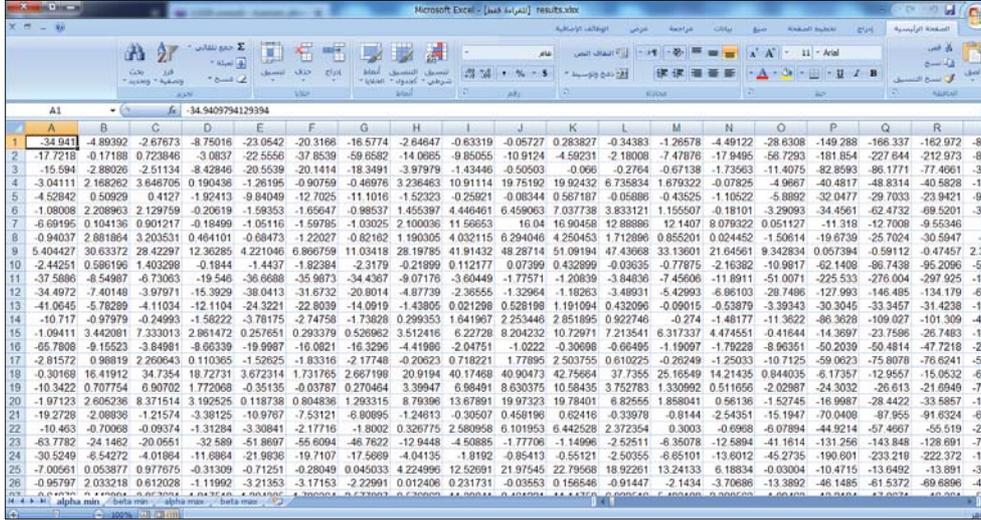


FIGURE 2. Daily mean of maximum temperature [°C]

$$f(x) = \frac{e^{-\frac{x-\mu}{s}}}{s \left(1 + e^{-\frac{x-\mu}{s}} \right)^2} \quad (1)$$

$-\infty < x < \infty$

where:

μ – location parameter,

s – scale parameter,

x – random variable.

Rayleigh distribution

Rayleigh distribution is a continuous probability distribution named after the English Lord Rayleigh. The distribution is widely used:

- In communications theory to model multiple paths of dense scattered signals reaching a receiver.
- In the physical sciences to model wind speed, wave heights and sound/light radiation.

- In engineering to measure the lifetime of an object, where the lifetime depends on the object's age, for example: resistors, transformers, and capacitors in aircraft radar sets.

- In medical imaging science to model noise variance in magnetic resonance imaging.

Rayleigh distribution is a special case of the Weibull distribution with a scale parameter of 2. When a Rayleigh is set with a shape parameter (σ) of 1, it is equal to a χ^2 distribution with 2 degrees of freedom.

The notation x Rayleigh means that the random variable x has a Rayleigh distribution with shape parameter (σ). The probability density function ($x > 0$) is:

$$\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad (2)$$

The expected value of a probability distribution is:

$$E(x) = \int xf(x)dx \quad (3)$$

Substituting in the Rayleigh probability density function, this becomes:

$$E[x] = \int_0^{\infty} x \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad (4)$$

This Wolfram calculator will solve the integral for you, giving the Rayleigh expected value of $\sigma\sqrt{\pi/2}$. The variance of a Rayleigh distribution is derived in a similar way, giving the variance formula of (Aslam, Tahir, Hussain & Al-Zahrani, 2015):

$$\text{var}(x) = \sigma^2 \left(\frac{4 - \pi}{2} \right) \quad (5)$$

Gamma distribution

Gamma distribution is a family of right-skewed, continuous probability distributions. These distributions are useful in real-life where something has a natural minimum of 0. For example, it is commonly used in finance, for elapsed times, or during Poisson processes.

Gamma distribution PDF

If x is a continuous random variable then the probability distribution function is:

$$f(x; \sigma; s) = \left\{ \frac{1}{(S^s \sigma \Gamma(\sigma))} x^{s-1} e^{-x/\sigma} \right\}$$

$$x \geq 0$$

(6)

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad (7)$$

where:

$\Gamma(x)$ – gamma function,

σ – shape parameter.

s (sometimes θ is used instead) – rate parameter (the reciprocal of the scale parameter).

Variables σ and s are both greater than 1. When $\sigma = 1$, this becomes the exponential distribution. When $s = 1$, this becomes the standard gamma distribution. Alpha and beta define the shape of the graph. Although they both have an effect on the shape, a change in s will show a sharp change (Artemiou, 2009).

Goodness-of-fit test

Goodness-of-fit tests are used to check the accuracy of the predicted data using theoretical probability function. To evaluate the goodness-of-fit of the PDFs to the monthly maximum temperature data, Z-test was used.

Z-test

The one-sample Z-test assumes that the data are a simple random sample from a population of normally distributed values that all have the same mean and variance (known). This assumption implies that the data are continuous and their distribution is symmetric. The calculation of the Z-test proceeds as follows (Weaver, 2011):

$$Z = \frac{X - A}{\frac{S}{\sqrt{n}}} \quad (8)$$

where:

X – mean of sample,

A – mean of population,

S – standard deviation of population,

n – number of observations.

Results and discussion

Basic statistics of maximum temperature for the study area for each month are shown in Table 1.

The estimation of parameters of all the PDFs considered in this study were carried out using maximum likelihood method and estimated parameters for the mean of each month of different PDFs used are presented in Table 2.

June, July and August was as follows: 0.119, 0.1395, 0.1399 (Figs. 3, 4).

The highest probability of an average maximum temperature in the Rayleigh distribution in March, April and May was as follows: 0.0289, 0.0331, 0.0201, in June, July and August was as follows: 0.0182, 0.0177, 0.0179 (Figs. 5, 6).

The highest probability of an average maximum temperature in the gamma distribution in March, April and May was

TABLE 1. Basic statistics of maximum temperature for all month at the study area

Month	Mean	Median	Maximum	Minimum	Standard deviation
	°C				
March	21.11	23.44	45.65	-16.43	1.41
April	22.03	25.49	36.30	-24.29	0.96
May	30.18	31.34	50.62	-23.99	1.10
Jun	34.49	35.21	52.63	5.89	0.92
July	35.67	36.20	53.79	8.83	0.84
August	36.32	36.33	54.70	8.39	0.94

TABLE 2. Parameters of different probability distribution

Probability distribution	Parameter	March, April, May	June, July, August
Logistic	location	25.76	33.9
	scale	3.00	1.91
Rayleigh	scale	25.76	33.9
Gamma	shape	80.1	321.8

By using STATSTIC program to estimate the probability density function was calculated using a number of distributions (logistic, Rayleigh and gamma distribution). Through the distributions the table was obtained.

In the logistic distribution the highest probability of an average maximum temperature in March, April and May was as follows: 0.077, 0.079, 0.093, in

as follows: 0.064, 0.049, 0.036, in June, July and August was as follows: 0.0259, 0.0219, 0.0211 (Figs. 7, 8).

Value of statistical test for different distribution of Middle East, test was performed between the calculated and tabulated value of each distribution as shown in Tables 3, 4 and 5 for all distributions (logistic, Rayleigh and gamma).

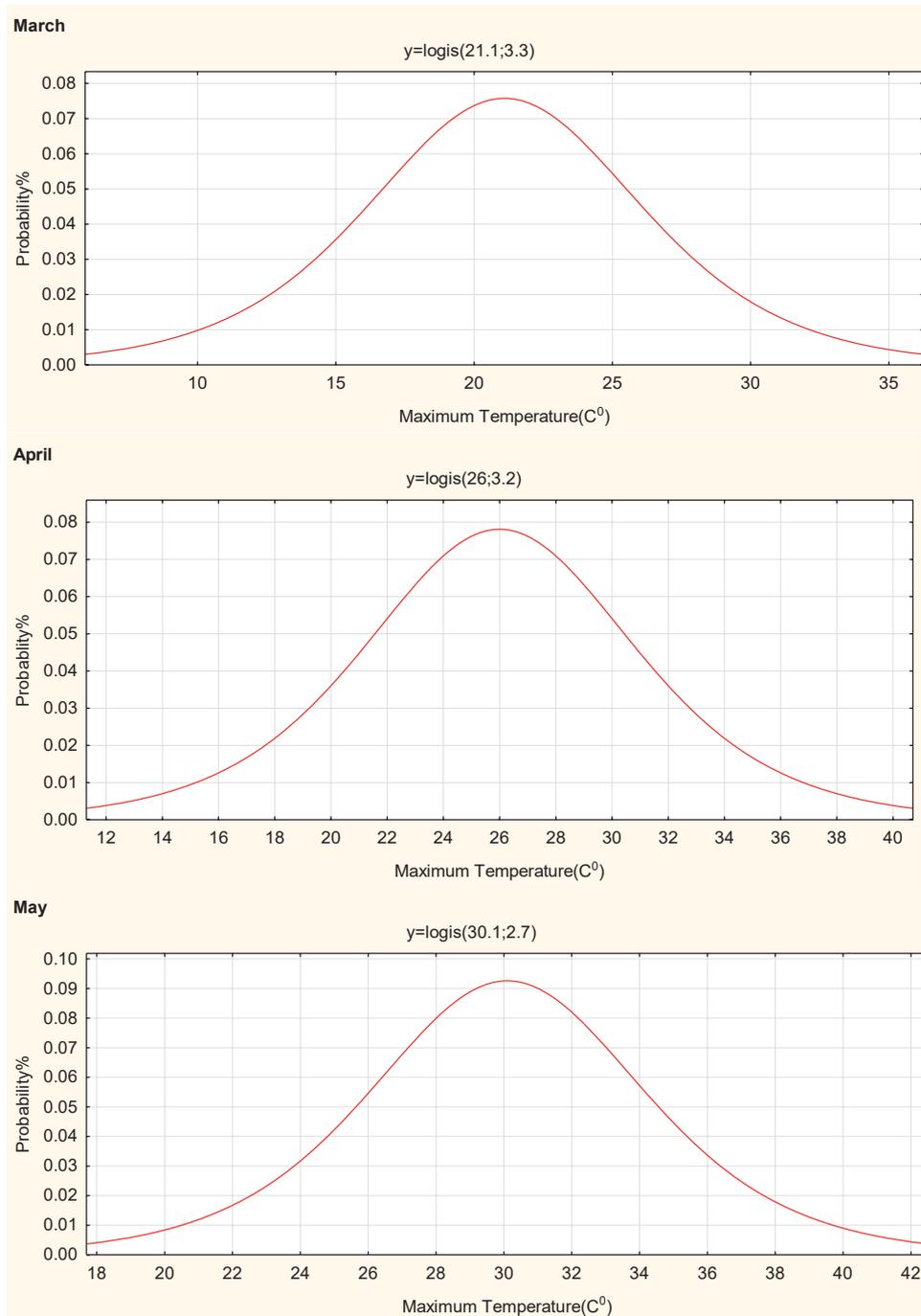


FIGURE 3. Logistic distribution for the maximum temperature in March, April and May at the study area

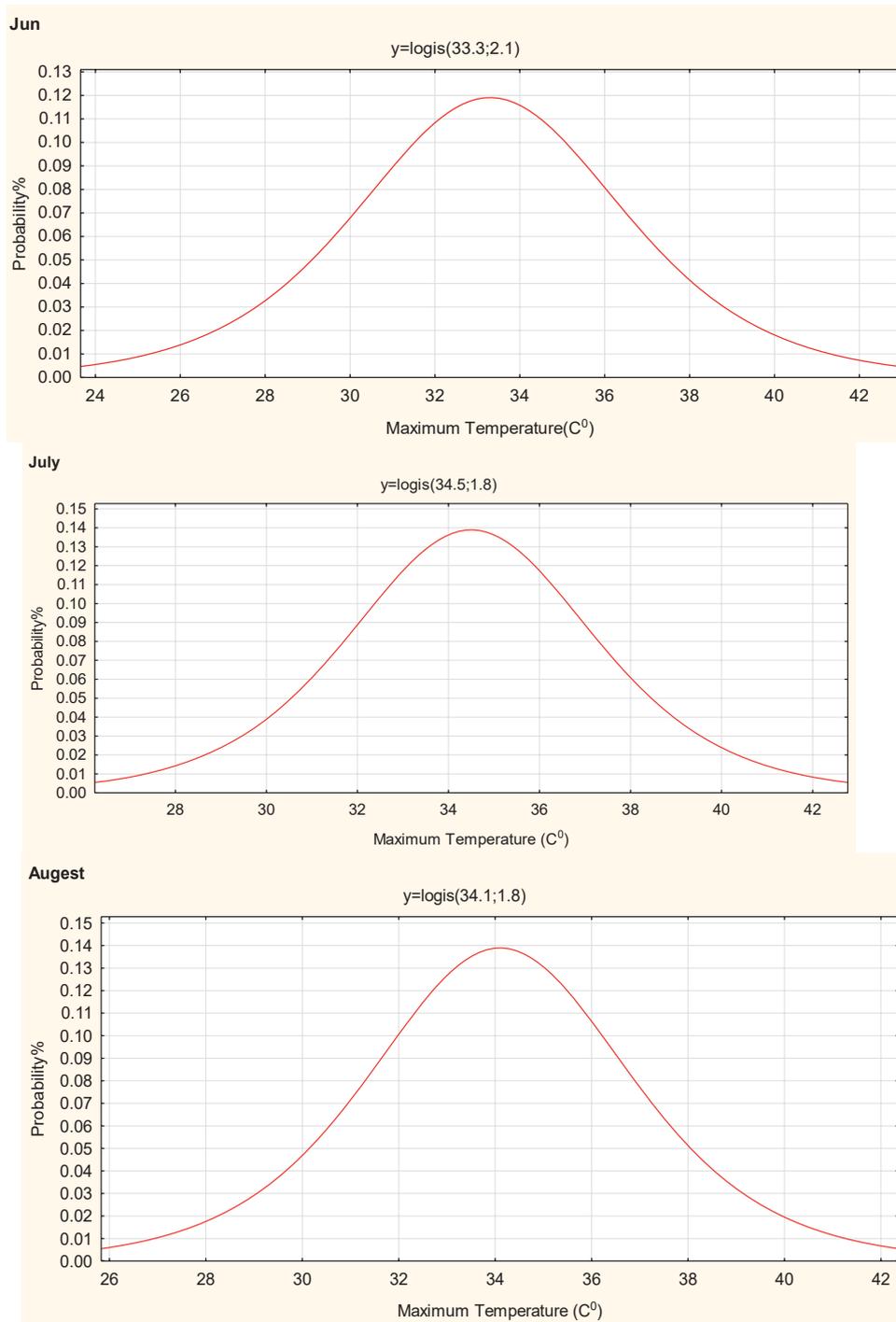


FIGURE 4. Logistic distribution for the maximum temperature in June, July and August at the study area

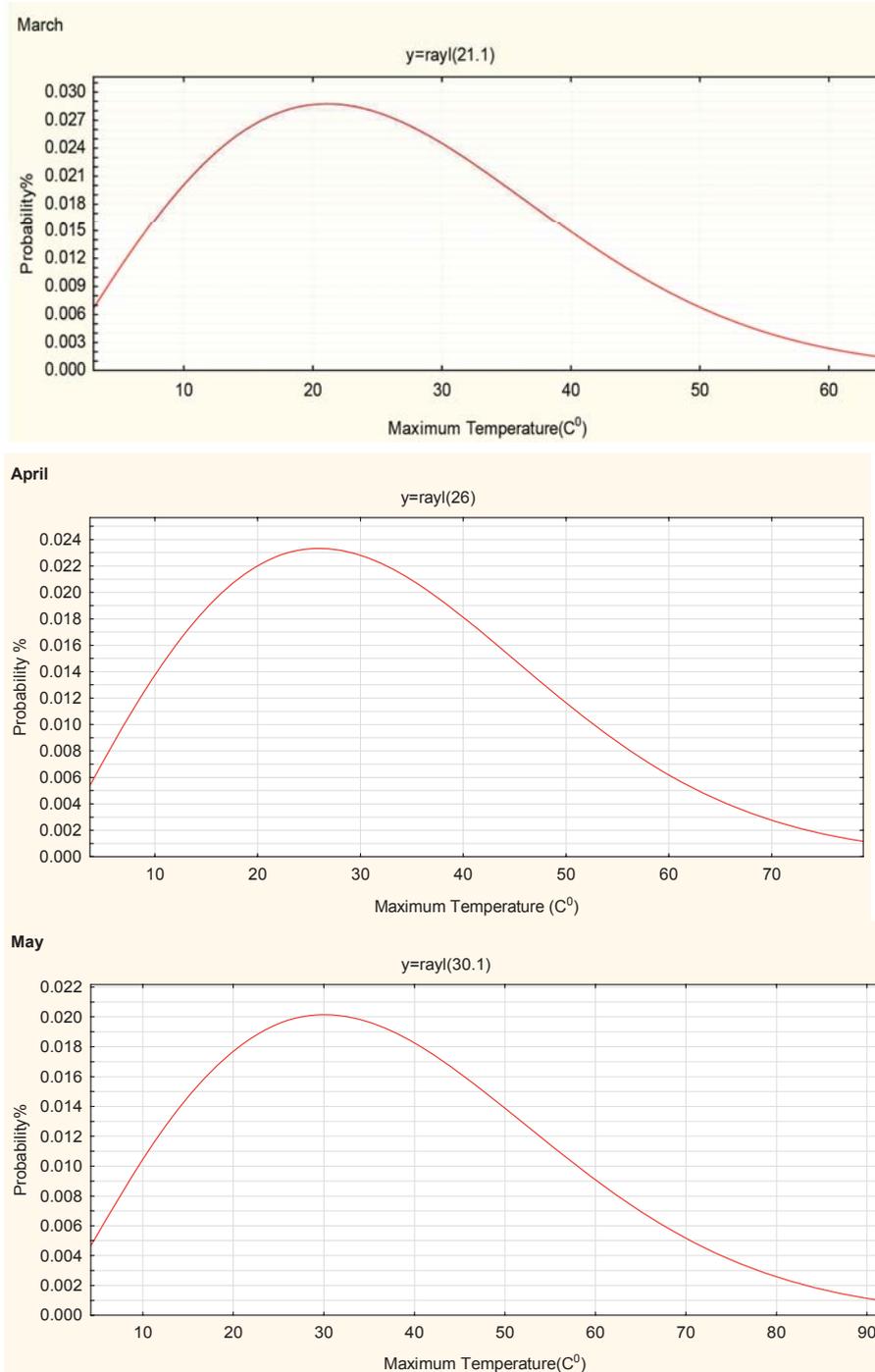


FIGURE 5. Rayleigh distribution for the maximum temperature in March, April and May at the study area

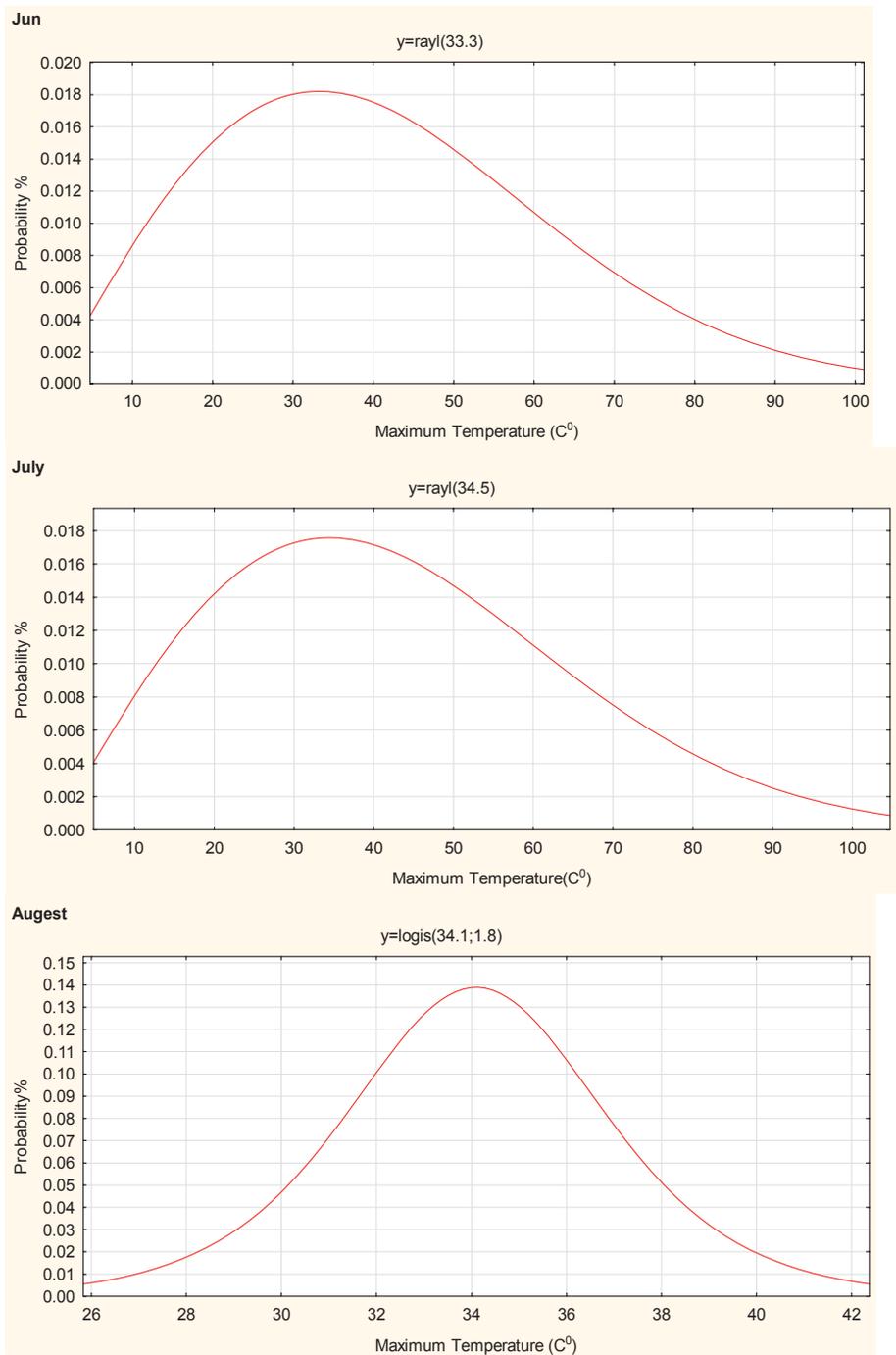


FIGURE 6. Rayleigh distribution for the maximum temperature in June, July and August at the study area

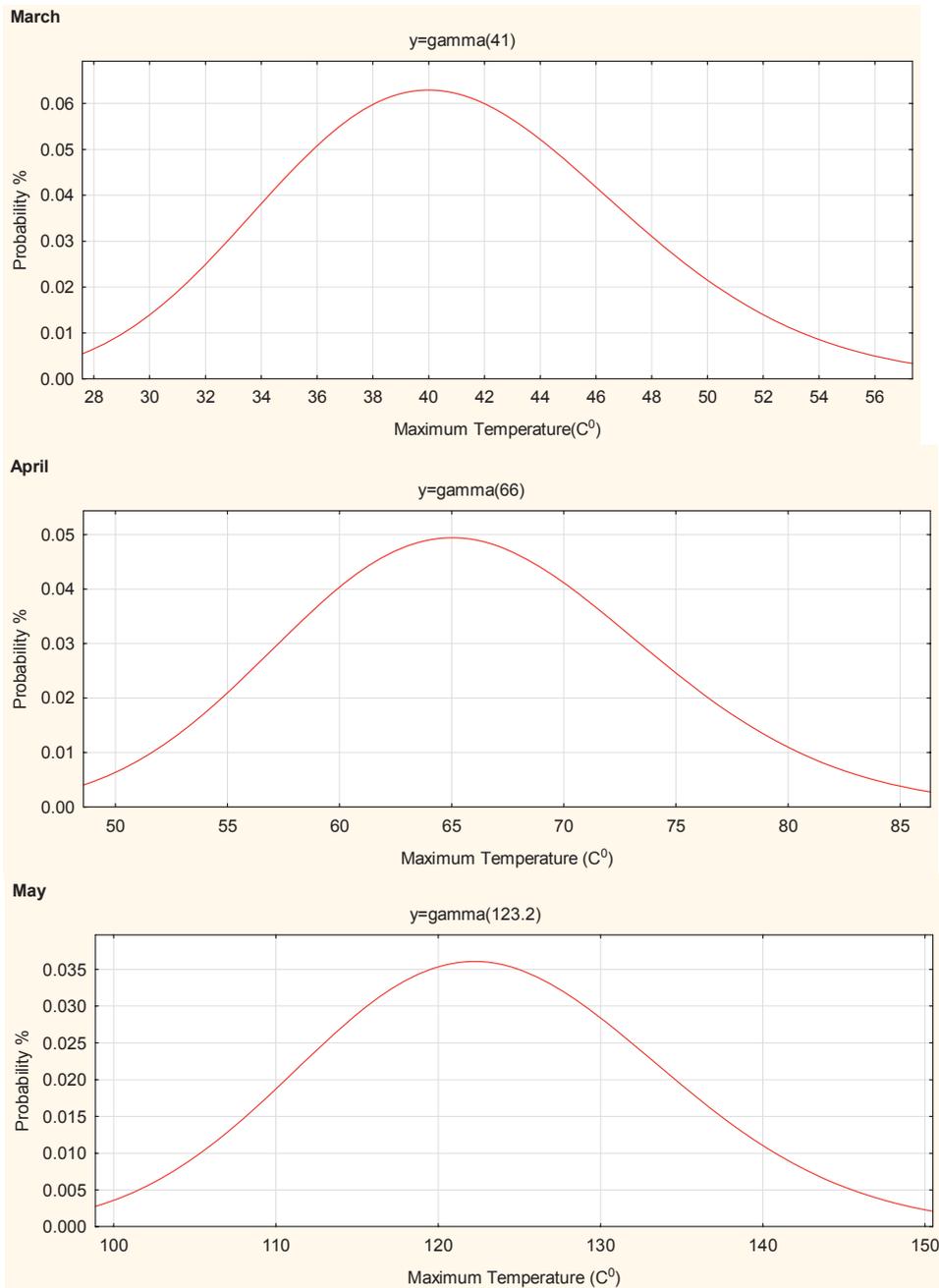


FIGURE 7. Gamma distribution for the maximum temperature in March, April and May at the study area

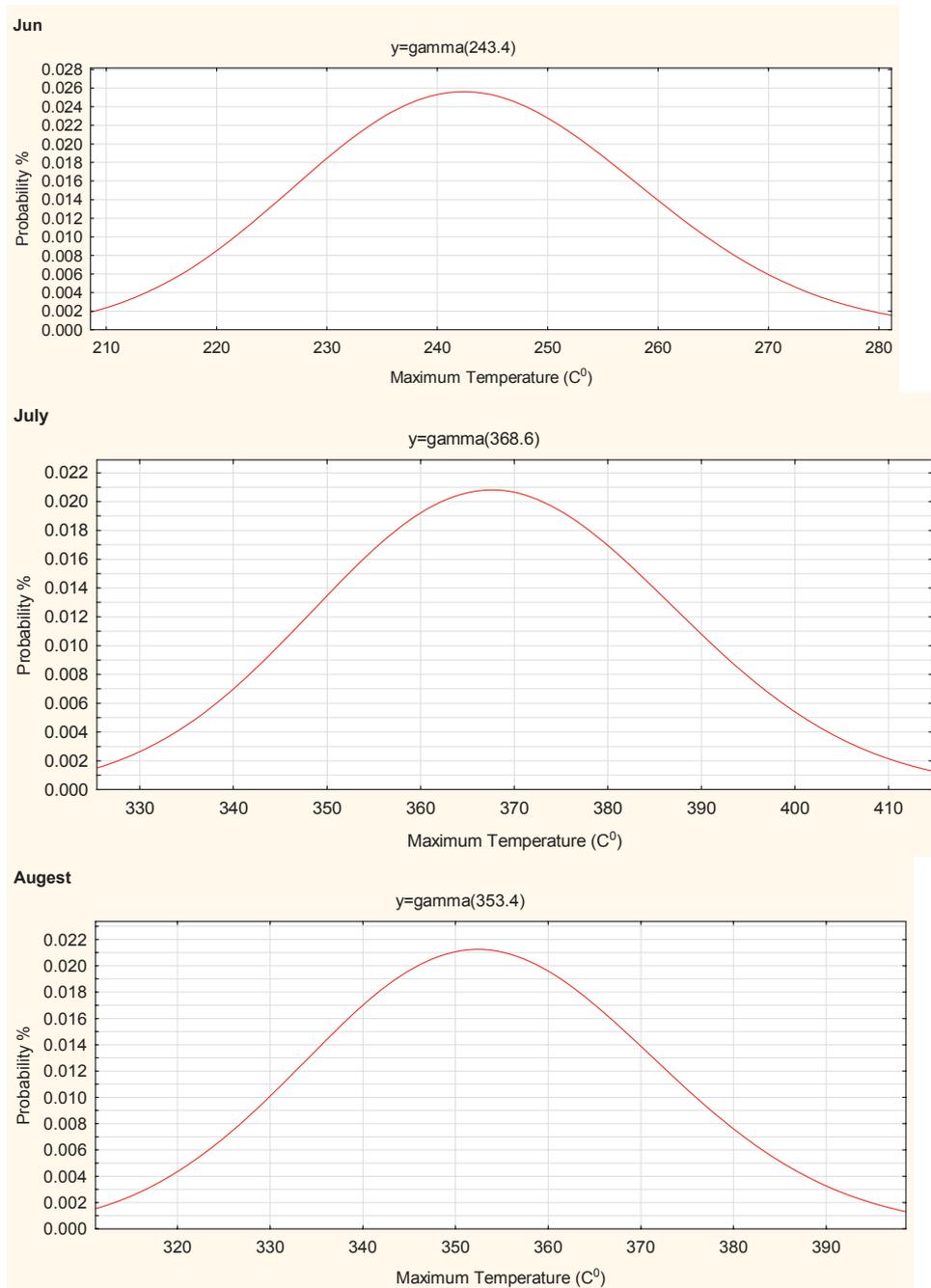


FIGURE 8. Gamma distribution for the maximum temperature in June, July and August at the study area

TABLE 3. Value of the calculated and tabulated probability by using logistic distribution for all months

Month	Tabulated Z	Calculated Z
March	0.9573	1.723
April	0.9429	1.58
May	0.9821	2.10
June	0.9778	2.01
July	0.9599	1.75
August	0.9941	2.52

TABLE 4. Value of the calculated and tabulated probability by using Rayleigh distribution for all months

Month	Tabulated Z	Calculated Z
March	0.9474	1.62
April	0.9306	1.48
May	0.9744	1.95
June	0.9713	1.90
July	0.9554	1.70
August	0.992	2.41

TABLE 5. Value of the calculated and tabulated probability by using gamma distribution for all months

Month	Tabulated Z	Calculated Z
March	0.6179	0.3030
April	0.6217	0.3210
May	0.6443	0.370
June	0.6808	0.476
July	0.7088	0.550
August	0.7291	0.610

Determine the null and alternative hypotheses null hypothesis: No difference in average and the mean of maximum temperature alternative hypotheses. There is a difference in average and the mean of maximum temperature, by using Eq. (8) for Z-test as shown in Tables

3, 4 and 5 for three probability distributions, find tabulated Z from Table 6. If the value of significance level 0.05, so tabulated $Z = 1.96$. Decision and interpretation: Therefore, we reject the null hypothesis.

Conclusions

Probability analysis of monthly daily mean of maximum temperature of Middle East was carried out by employing three probability distributions namely logistic, Rayleigh and gamma distribution. Goodness of fit of these distributions was tested by Z-test for monthly daily mean of maximum temperature. Logistic's distribution is found the best fit distribution. By comparing the value that was extracted for each of the summer and spring months with the tabular values, it was found that the calculated values are greater than the calculated values and when compared with the tabular values, it was found that the table values for summer months are greater than the tabular values of the spring months, therefore, we reject the null hypothesis and accept the alternative hypothesis, and this is concluded in this research through this distribution. This same statement applies to the other two distributions: Rayleigh and gamma distribution (Bhakar, Iqbal, Devanda, Chhajed & Bansal, 2008; Waterson, 2008).

References

- Al-Kadim, K.A. (2011). On characterization of the logistic distribution. Retrieved from: <https://tinyurl.com/y2qzh6e5>

TABLE 6. Table of the normal distribution



**Probability Content
from -∞ to Z**

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9978	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Artemiou, A. (2009). *Gamma distribution and its relatives*. Retrieved from: http://www.math.mtu.edu/aatemio/Courses/Stat318/Lectures/Chater4_Lecture4.pdf

Aslam, M., Tahir, M., Hussain, Z. & Al-Zahrani, B. (2015). A 3-Component Mixture of Rayleigh Distributions: Properties and Estimation in Bayesian Framework. *PLOS ONE*, 10(5), e0126183. <https://doi.org/10.1371/journal.pone.0126183>

Bhakar, S.R., Iqbal, M., Devanda, M., Chhajed, N. & Bansal, A.K. (2008). Probability analysis of rainfall at Kota. *Indian Journal of Agricultural Research*, 42(3), 201-206.

Brown, O. & Crawford, A. (2009). *Rising tension, climate change and the risk of violent conflict in the Middle East*. Winnipeg: International Institute for Sustainable Development.

Evans, J.P., Smith, R.B. & Oglesby, R.J. (2004). Middle East climate simulation and dominant precipitation processes. *International Journal of Climatology*, 24(13), 1671-1694.

Hasanean, H.M. (2015-08-07). Tropical meteorology – Middle East meteorology. In *Encyclopedia of Life Support System*. UNESCO-EOLSS. Retrieved from: <https://www.eolss.net/Sample-Chapters/C01/E6-158-19.pdf>

Karl, T.R., Melillo, J.M. & Peterson, T.C. (2009). *Global climate change impacts in the United States*. Cambridge: Cambridge University Press.

Paul, P. (2013). *ECMWF reanalysis: Resources for the wind energy community and a few global reanalysis myth-busters*. 2013 EWEA Technology Workshop Resource Assessment, Dublin, Ireland. ECMWF. Retrieved from: <http://www.ewea.org/events/workshops/wp-content/uploads/2013/06/EWEA-RA2013-Dublin-1-1-Paul-Poli-ECMWF.pdf> [slideshow].

Steffen, W., Hughes, L. & Perkins, S. (2014). *Heat waves: hotter longer, more often*. Sydney: Climate Council of Australia.

Tanarhte, M., Hadjinicolaou, P. & Lelieveld, J. (2015). Heat wave characteristics in the Eastern Mediterranean and Middle East using extreme value theory. *Climate Research*, 63(2), 99-113.

Watterson, I.G. (2008). Calculation of probability density functions for temperature and precipitation change under global warming. *Jour-*

nal of Geophysical Research, 113, D12106.
<https://doi.org/10.1029/2007JD009254>

Weaver, B. (2011-05-27). *Hypothesis testing using z- and t-tests*. Retrieved from: http://www.angelfire.com/wv/bwhomedir/notes/z_and_t_tests.pdf

Zhang, X., Aguilar, E., Sensoy, S., Melkonyan, H., Tagiyeva, U., Ahmed, N., Kutaladze, N., Rahimzadeh, F., Taghipour, A., Hantosh, T.H., Albert, P., Semawi, M., Karam, A.M., Al-Shabibi, M.H.S., Al-Oulan, Z., Zafari, T., Khelet, I.A.D., Hamoud, S., Sagir, R., Demircan, M., Eken, M., Adiguzel, M., Alexander, L., Peterson, T.C. & Wallis, T. (2005). Trends in Middle East climate extreme indices from 1950 to 2003. *Journal of Geophysical Research*, 110, D22104. <https://doi.org/10.1029/2005JD006181>

Summary

Estimate the probability density function of maximum temperature for the Middle East. Pollution is one reasons for increase temperature which leads to increase the heat waves which have large socio-economic and healthy impacts on Middle East. By using monthly daily mean of maximum temperature at height of 2 m covered Middle East as a grid of 1,581 points for selected months (March, April, May) represent spring and June, July, August represent summer for

the period 1979–2018, from the ECMWF, model ERA-interim. Many PDFs have been proposed in recent past, but in present study logistic, Rayleigh and gamma distribution are used to describe the characteristics of maximum temperature. This paper attempts to determine the best fitted probability distribution of maximum temperature. To check the accuracy of the predicted data using theoretical probability distributions the goodness of fit criteria Z-test used in this paper. According to the goodness-of-fit criteria and from the graphical comparisons it can be said that logistic distribution provides the best fit for the observed monthly daily mean of maximum temperature data.

Authors' address:

Iqbal K. Al-Ataby – corresponding author
(<https://orcid.org/0000-0002-0111-9324>)
Mustansiriyah University
College of Science
Atmospheric Sciences Department
Baghdad, Iraq
e-mail: iqbsl.atmsc@uomustansiriyah.edu.iq

Amani I. Al-Tmimi
(<https://orcid.org/0000-0002-7314-6057>)
Al-Karkh University of Science
Energy and Environment Sciences College
Baghdad, Iraq
e-mail: dr.amani_almimi@kus.edu.iq