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Mathematical model for the capacity of the mud flow with wave regime taking into account its rheological properties

Keywords: mudflow, power, wave motion, rheology, energy properties

Introduction

The process of driving a mudflow mass originating from a mudflow center and the prediction of the expected value is a complex problem.

Therefore, as soon as the hydrological, hydraulic and energy characteristics of the mudflow are known, the choice of engineering measures for flood control is no longer difficult.

Recently, frequent impact of mudflows on the environment has become intense. Frequent occurrences of incitement of mudflow masses in the mudflow centers and impacts on facilities have been identified. The process corresponding to the onset

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of movement from the epicenter is very complex. Therefore, it is difficult to predict the volume and discharge of alluvial mass driven from the erosion center.

As it is confirmed by the statistics of studying the alluvial processes, the formation of a mudflow from erosion centers and the beginning of movement occurs as a continuous dynamic or monoclinal wave. Continuous or stepwise changes in hydraulic and hydrological parameters of the flow are typical to the cases of mudflow movement as a wave, accompanied by a gradual process of "increase--and-decrease" of energy parameters, in particular discharge, velocity and levels, as well as cases of discontinuous changes or wave-like motion (Gavardashvili, 2022; Gavardashvili et al., 2023).

Flood formation as a wave is always associated with the violation of the equilibrium state of the mudflow forming mass in the center, overcoming the barriers of the encountered resistances in the transit zone, or impulses caused by the influence of other external forces. A mudflow moves as a continuous wave when the current is transformed from one stationary state to another. The development of the motion process in this way is similar to a quasi-stationary event, and this is to be expected when gravitational forces are gradually balanced by resistance forces.

Material and methods

The disruption of the deformed state of the mudflow mass formed in the erosion center corresponds to a certain phase of the equilibrium state of its body and is associated with certain limits of the mutual ratio of the constituent components. Accordingly, the violation of equilibrium and the beginning of movement are related to the value of the active transverse force, and the dynamics of movement is related to the filling of the pores of the mudflow mass with water, capillary moisture, effective flow and cohesion forces.

In order to obtain the calculation equation for the power of the mudflow mass, the limit stress state of the solid inert mass accumulated in the erosive banks as a result of negative geomorphological processes on the mountain slopes (erosion of the mountain slopes, landslides, rock falls, snow avalanches, etc.) is considered, using the basic equations of ground mechanics and hydrodynamics.

In connection with the above, the study of the tensed state of the mudflow mass can be considered analogous to the problems of soil mechanics (Fig. 1).

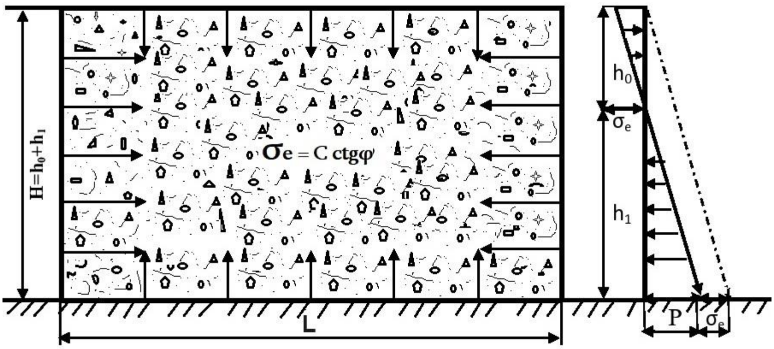

FIGURE 1. Calculation scheme
Source: own work.

When the density of the mudflow-forming mass (ρ), the gravity acceleration (g), the angle of internal friction (φ), the height of layer (h') of adherence pressure (Pe), the height of the mudflow-forming mass (H), the intensities of transverse pressure (P), and cohesiveness (C) are known, the value of active pressure is (Gregoretti et al., 2011; Natishvili et al., 2020):

$$P = \rho \mathrm{g}\left(H + h'\right)\mathrm{tg}^2\left(45° - \frac{\phi}{2}\right) - Pe, \tag{1}$$

when the value of cohesion pressure: $Pe = \gamma h' = pgh'$ and the equivalent depth of cohesion: $' \frac{\quad}{\rho \mathrm{g}\,\mathrm{tg}\,\phi}$, the value of active pressure is calculated by the following relation (Kailey et al., 2011):

$$P = \rho \mathrm{g} H\,\mathrm{tg}^2\left(45° - \frac{\phi}{2}\right) - 2C\,\mathrm{tg}^2\left(45° - \frac{\phi}{2}\right). \tag{2}$$

Based on the principle of inert mass tension, the effect of the active pressure in the cohesive mudflow-forming mass on the ultimate resistance starts at a certain depth from its surface. Hence, when $P = 0$, $H = h_0$, we will have:

$$h_0 = \frac{2C}{\gamma\,\mathrm{tg}\left(45° - \frac{\phi}{2}\right)}, \tag{3}$$

where h_0 is the depth, corresponding to the zero value of the tension.

If we introduce notation $\mathrm{tg}^2\left(45° - \frac{\phi}{2}\right) = \psi$, the maximum tension distribution in the stressed zone is calculated:

$$P = \gamma H\left(1 - \frac{h_0}{H}\right)\psi \tag{4}$$

and the depth corresponding to the stressed state is calculated:

$$\frac{P}{\gamma} = h_1\left(1 - \frac{h_0}{H}\right)\psi. \tag{5}$$

Based on the active pressure, the magnitude of its corresponding transverse force is:

$$P = \frac{\gamma H^2}{2}\left(1 - \frac{h_0}{H}\right)^2 \psi. \tag{6}$$

The active force *P* is the force destructing the equilibrium resistance of the mudflow mass and the volume of the mudflow mass moved under its action when the width of its hearth is *B*. In this case:

$$W = \frac{P}{\gamma} = \frac{BH^2}{2}\left(1 - \frac{h_0}{H}\right)^2 \psi, \tag{7}$$

when the length of the hearth is *L*, the volume of the mudflow mass moved is:

$$BHL = \frac{BH^2}{2}\left(1 - \frac{h_0}{H}\right)^2 \psi. \tag{8}$$

Accordingly, the length of the active force exposure is equal to:

$$L = \frac{H}{2}\left(1 - \frac{h_0}{H}\right)\psi \tag{9}$$

and the active height of mudflow's hearth is: $H_a = H\left(1 - \frac{h_0}{H}\right)\psi$.

The disturbance of the equilibrium state of the mudflow mass in the erosion center and the value of the discharge of the onset of movement (Kaitna et al., 2011) is:

$$Q = \frac{W}{t}. \tag{10}$$

If we introduce the value of Eq. (10) into Eq. (8), we get:

$$Qt = \frac{BH^2}{2}\left(1 - \frac{h_0}{H}\right)^2 \psi. \tag{11}$$

For the ratio of unit width and length of the mudflow-forming center $Q = q'_{nc}$. Consequently:

$$q'_m t = H\left(1 - \frac{h_0}{H}\right)\psi. \tag{12}$$

The relationship between the discharge and intensity of possible mudflow that started from the erosion center is:

$$q'_m = \frac{2Q}{H\left(1 - \frac{h_0}{H}\right)\psi}. \tag{13}$$

If we differentiate equation (12) with limit condition $q'_{nc} = const$, we will obtain:

$$q'_m dt = \left(1 - \frac{h_0}{H}\right)\psi dH, \tag{14}$$

when the resistance coefficient of the mudflow mass that started from the center is a constant value and $\varphi = 1$ and $h_0 = 0$, then:

$$q'_m = \frac{dH}{dt}. \tag{15}$$

Hence, by integrating (15), we obtain:

$$q'_m\left(t - t_0\right) = H - H_0 \tag{16}$$

and

$$q'_m\left(1-\frac{h_0}{H}\right)=\psi q'_n. \tag{17}$$

If we substitute the value of Eq. (16) into Eq. (17), we obtain:

$$q'_m\left(t-t_0\right)=\left(1-\frac{h}{H}\right)\psi\left(H-H_0\right). \tag{18}$$

The occurrence of mudflow movement onset from the erosion center is described by Eq. (14) as a wave what can be represented as follows:

$$q'_m\left(1-\frac{h_0}{H}\right)\phi\frac{dHdx}{dx\cdot dt}=\left(1-\frac{h_0}{H}\right)\psi V_B\frac{dH}{dx}. \tag{19}$$

In the case of uniform steady motion, the mudflow discharge is a function of its depth. In the case of motion as a continuous wave, the wave velocity, following the continuity condition, is three times its mean velocity $\overline{V}$ and accordingly, by integrating the nineteenth equation including mathematical transformations, we will get (Arnold et al., 2011):

$$q'_m=3\left(1-\frac{h_0}{H}\right)\psi\overline{V}\frac{dH}{dx}=H-H_0. \tag{20}$$

Based on the generalized Shvedov–Bingham model, the value of the average velocity is calculated by the following relation (Steger & Warming, 1981):

$$\overline{V}=\frac{\gamma iH^2}{3\mu}\left(1-\frac{h_0}{H}\right)^2\psi\left(1+\frac{1h_0}{2H}\right). \tag{21}$$

By inserting the value of Eq. (22) into the relation (21) and transforming it, we obtain:

$$q'_{nc}=\left(1-\frac{h_0}{H}\right)^3\phi^2\frac{\gamma iH^2}{\mu}\left(1-\frac{1h_0}{2H}\right)\frac{dH}{dx}. \tag{22}$$

Based on the simplification and transformation of Eq. (23), when $\beta = \frac{h_0}{H}$ and $f(\beta) = (1-\beta)^2 \left(1 + \frac{1h_0}{2H}\right)\psi$, we obtain:

$$q'_m = \frac{giH^2}{\nu}\left(1 - \frac{h_0}{H}\right)\psi\, f(\beta)\frac{dH}{dx}. \tag{23}$$

For a particular mudflow wave when the 0X axis coincides with direction of mudflow motion, by integrating Eq. (23), we obtain:

$$\frac{\left(H^3 - H_0^3\right)\left(1 - \frac{h_0}{H}\right)\psi\, f(\beta)\,gi}{\nu} = q'_m\left(x - x_0\right). \tag{24}$$

The resulting formula (24) is the equation of the free surface of the wave in plane *Hx*. If calculating the value of the mudflow depth *H* by the relation (23), we obtain:

$$\left[H_0 + \frac{q'_m\left(t - t_0\right)}{\left(1 - \frac{h_0}{H}\right)\psi}\right]^3 = H_0^3 + \frac{q'_m\left(x - x_0\right)\nu}{gif(\beta)\left(1 - \frac{h_0}{H}\right)\psi}. \tag{25}$$

If excluding H_0 from the relation (24), the value *H* can be determined in time *t* and the profile of the free surface of the wave can be described by the following equation:

$$H^3 = \left[H - \frac{q'_m\left(t - t_0\right)}{\left(1 - \frac{h_0}{H}\right)\psi}\right]^3 = \frac{q'_m\left(x - x_0\right)\nu}{gif(\beta)\left(1 - \frac{h_0}{H}\right)\psi}. \tag{26}$$

In the case of starting the mudflow motion from the initial position when $x = 0$, and $t_0 = 0$, in the case of different values of *H* at the first mudflow wave formation, from Eq. (24) we have:

$$H^3 = H_0^3 + \frac{q'_m x\nu}{gif(\beta)\left(1 - \frac{h_0}{H}\right)\psi}. \tag{27}$$

The propagation length of the wave in the plane *xt* can be determined from equation:

$$\left[H_0 + \frac{q'_m t}{\left(1-\frac{h_0}{H}\right)\psi}\right]^3 = H_0^3 + \frac{q'_m x \nu}{gif\left(\beta\right)\left(1-\frac{h_0}{H}\right)\psi}. \tag{28}$$

The profile of the wave surface $x = 0$, and $t = 0$, is determined by Eq. (26):

$$H^3 = \left[H - \frac{q'_m t}{\left(1-\frac{h_0}{H}\right)\psi}\right]^3 + \frac{q'_m x \nu}{gif\left(\beta\right)\left(1-\frac{h_0}{H}\right)\psi}. \tag{29}$$

At a later time moment when $x = 0$ and $H_0 = 0$, the wave surface profile is defined by equation:

$$H_0^3 = \frac{q'_m x \nu}{gif\left(\beta\right)\left(1-\frac{h_0}{H}\right)\psi}. \tag{30}$$

Accordingly, the wavelength of wave propagation is calculated by Eq. (24) and the time *t* is calculated by the following equation:

$$t = t_0 + \sqrt[3]{\frac{x\nu\left(1-\frac{h_0}{H}\right)^2\psi}{q^2{}_m gif\left(\beta\right)}}, \tag{31}$$

where *i* is the gradient of the uniform motion of mudflow.

In the first approximation, it is possible to analyze the process of mudflow formation from erosion foci. By means of the methodology used and the obtained Eq. (27), the formation of micropower floods can be predicted.

For the first destructive mudflow wave when $t = 0$ and $x = 0$, the wave propagation length can be described by equation, and for microwaves in plane *xt* when $H_0 = 0$, $x = 0$, and $t_0 = 0$, it is calculated by Eq. (31).

Assessment of stability violation of the mudflow mass formed in the erosion center, when on the basis of the theory of stresses the beginning of movement and movement of one part of the formation onto another is predicted, the discharge change dynamics during the movement is non-stationary. The mathematical model of mass movement in an erosion center corresponding to the reduction of its depth H to h can be determined by relating the mudflow formed by the active force to the stability characteristics.

Based on the natural reality, due to the multicomponent nature of the mudflow mass formed in the erosion center and different effects of weather and climatic factors as confirmed by the real event, the value of the discharge formed by the active lateral force is variable.

To evaluate the process of the motion onset of the mudflow mass and to represent the shear surface in a nonlinear form, the length of active wave propagation in relation to rheology can be calculated by Eq. (28).

In the case of using Eq. (24) to estimate the wave surface profile, when the characteristics of initial conditions are 0 and, accordingly, the change in its depth in relation to the intensity of the discharge change is represented by the dependence (27), the mudflow discharge for center height H_k can be determined by the formula:

$$Q = \frac{\mathrm{g}i\left(1-\frac{h_0}{H_k}\right)\psi f(\beta) H_k^3}{L}. \tag{32}$$

In the wave zone, when the magnitude of energy loss is a function of rheological characteristics, the relationship from depth H_k to h value can be defined as follows:

$$h = \frac{4H_k}{\left[2+\left(1-\frac{h_0}{H_k}\right)\psi\right]^2}. \tag{33}$$

Substituting the ratio (33) into Eq. (32), we obtain:

$$Q = \frac{\mathrm{g}i\left(1-\frac{h_0}{h}\right)\psi f(\beta)\left[1+\left(1-\frac{h_0}{H}\right)\psi\right]^6 h^3}{L}. \tag{34}$$

Based on the deduced dependence to calculate the discharge, the mudflow capacity is calculated by the following ratio:

$$N = \gamma Q H_k = \frac{4\gamma g i\left(1-\frac{h_0}{h}\right)\psi f(\beta)\left[1+\left(1-\frac{h_0}{H}\right)\psi\right]^6\left[2+\left(1-\frac{h_0}{h}\right)\psi\right]^2 h^4}{L}. \quad (35)$$

Based on the obtained ratio (35), it is possible to estimate the total capacity of the mudflow mass accumulated in erosion gullies, taking into account the main rheological properties of the solid mass eroded from the mountain slopes.

Conclusion

By taking into account the dynamics of wave motion of mudflows and rheological properties of the solid mass eroded from the mountain slopes, on the basis of theoretical studies, the equation to calculate the full capacity of the mudflow mass is deduced in the present paper.

When calculating the mudflow strength, attention is focused on the assessment of stability violation of the mudflow mass accumulated in the center of erosion when the onset of the mudflow mass movement predicted on the basis of the stress theory is non-stationary.

The derived formula (35) is currently being tested for particular cases of mudflow mass accumulated in erosion gullies identified in the river catchment basins in natural conditions.

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Summary

Mathematical model for the capacity of the mud flow with wave regime taking into account its rheological properties. The paper presents particular issues of mudflow dynamics, one of the hazardous natural disasters, namely the theoretical study of the flow power during mudflow movement in the wave regime taking into account its rheological properties. The paper discusses the physical process of mudflow mass impetus accumulated in erosion banks, taking into account the impact of the tense state of the eroded mass, in particular, similar to soil mechanics problems, the density of the mudflow-forming mass (ρ), the free fall acceleration (g), angle of internal friction (φ), the adhesive force (Pe), the height equivalent to pressure (h'), the height of the mudflow-forming mass (H), the intensity of transverse pressure (P), and the value of the active pressure of the inertial mass cohesion (C) on the deformation mode of the mudflow mass. On the basis of the basic equations of mudflow dynamics and theoretical studies, an equation is obtained to calculate the values of flow power when mudflows move in the wave mode, taking into account the main rheological properties of a mudflow mass.